

Musterprüfung

Themen:

- Betragsfunktion
- Betragsgleichungen

1.) Wie gross ist

a) $|4|?$

b) $|-3|?$

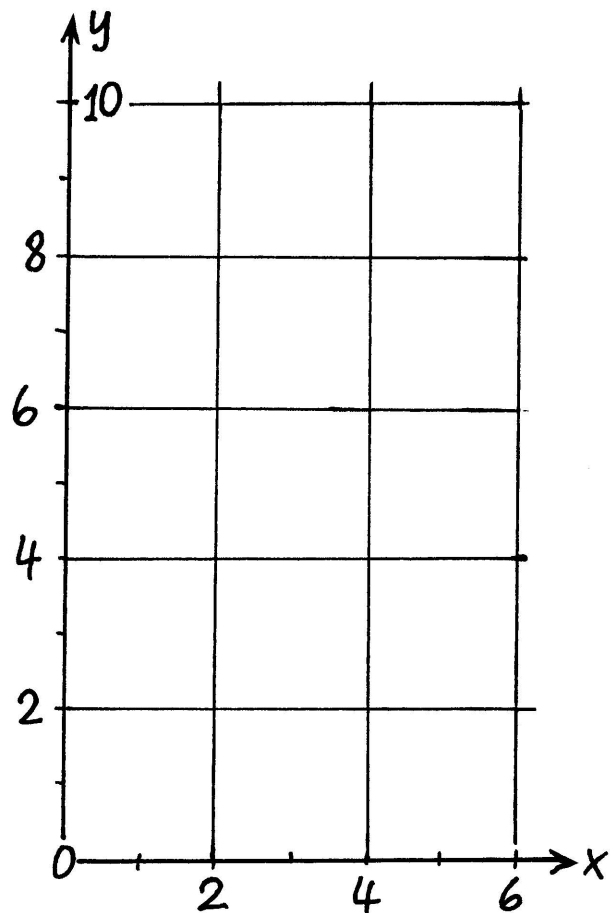
c) $|4 - |-4||?$

d) $x + |x - 4|?$

e) $|y - x|?$

2.) Vervollständige untenstehende Wertetabelle und skizziere die Funktion, wenn $y(x) = |x + |x - 2||$

x	y
0	
1	
2	
3	
4	
5	
6	



3.) Berechne die Lösungsmenge von

a) $x = |4|$

b) $|x| = 4$

c) $x = |-4|$

d) $|x| = -4$

e) $x = |x|$

f) $|x| + x = 4$

g) $|x| - x = 4$

h) $|x+3| = 2x-1$

i) $|3x-2| = 2x+12$

j) $|5x-8| = 3x-16$

k) $|x+11| = 3x-13$

4.) Bestimme die Lösungsmenge von

a) $|2x| + |x| = 12$

b) $|2x + |x|| = 12$

c) $|x+11| + |x+5| = 10$

d) $|x+10| - |x-10| = 10$

e) $2 \cdot |2x-3| - |x+3| = 18$

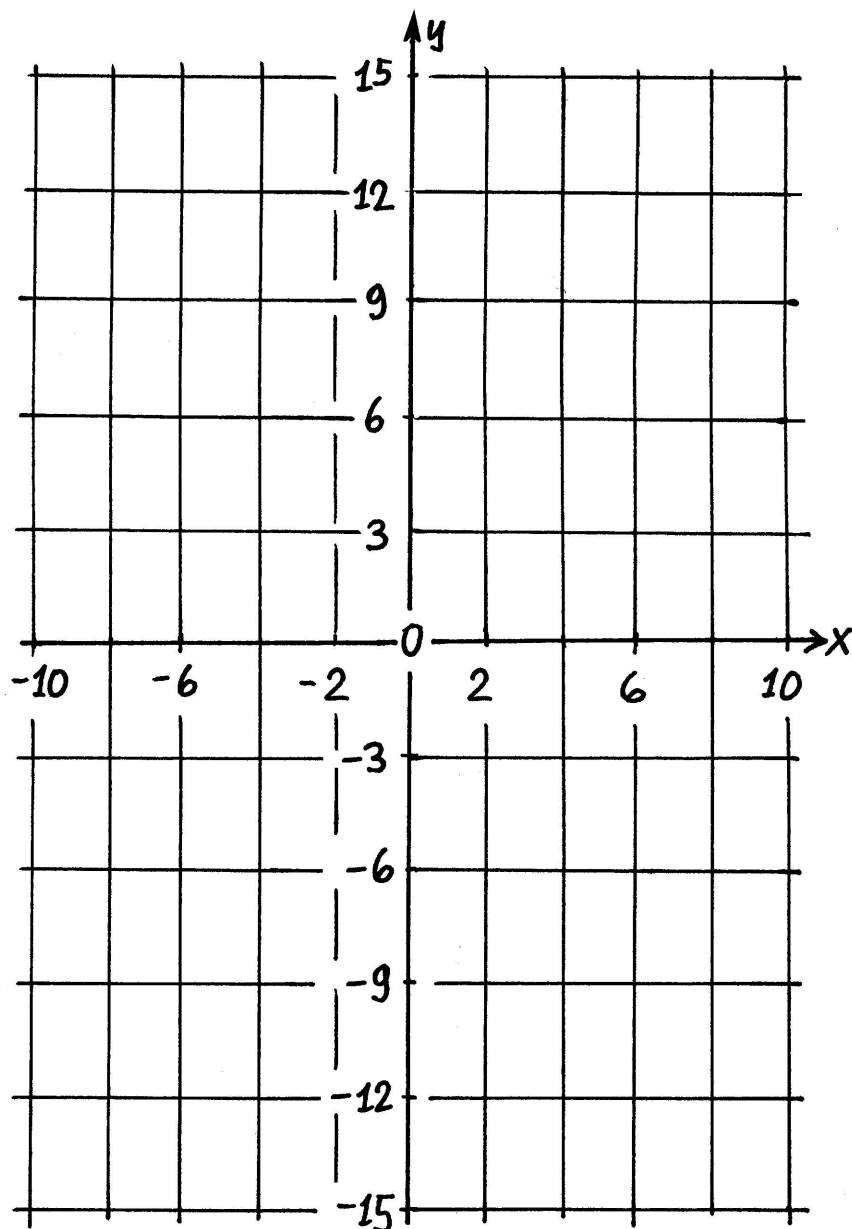
f) $(x+1) \cdot |x-1| = 8$

g) $\left| \frac{x+12}{x} \right| = 3$

h) $2 \cdot |2x+4| = 3 \cdot |x+9|$

5.) Für einen gegebenen Wert von x gibt es für die Abbildungsvorschrift $|x+y|=5$ zwei Werte für y . Vervollständige die Wertetabelle und skizziere die Lösungsmenge der Abbildungsvorschrift.

x	y ₁	y ₂
-10		
-8		
-6		
-4		
-2		
0		
2		
4		
6		
8		
10		



Lösungen

1a) 4

b) 3

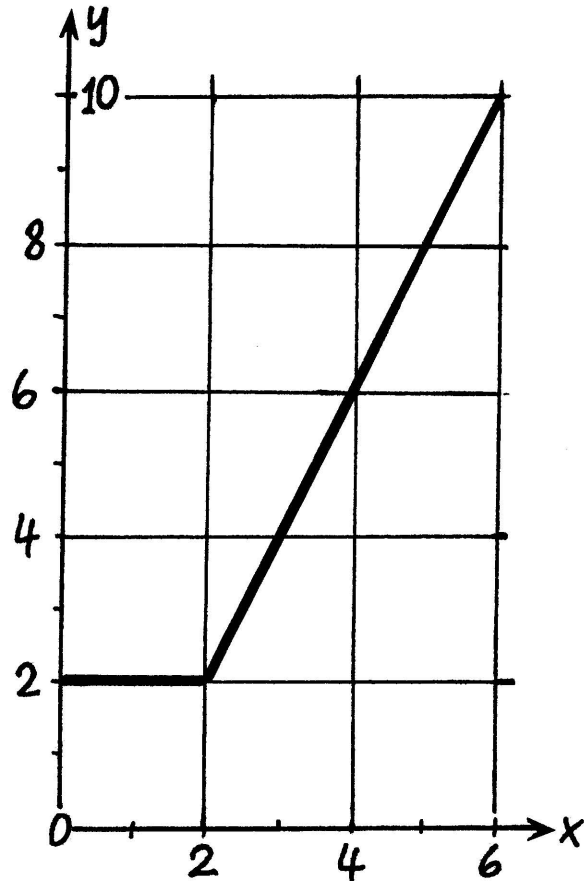
c) 0

$$b) x + |x - 4| = \begin{cases} 4 & \text{falls } x \leq 4 \\ 2x - 4 & \text{falls } x > 4 \end{cases}$$

$$c) |y - x| = \begin{cases} y - x & \text{falls } y \geq x \\ x - y & \text{falls } x < y \end{cases}$$

2.) $y(x) = |x + |x - 2||$

x	y
0	2
1	2
2	2
3	4
4	6
5	8
6	10



3a) $x = 4$

b) $x = \pm 4$

c) $x = 4$

d) Keine Lösung!
 $\mathbb{L} = \{\}$

e) $\mathbb{L} = \mathbb{R}_0^+$

$\mathbb{L} = \{x \in \mathbb{R} \mid x \geq 0\}$

f) 1. Fall: $x + x = 4 \rightarrow x_1 = 2$

2. Fall: $-x + x = 0 = 4 \rightarrow$ keine Lösung

Probe: $x_1 = 2: |2| + 2 = 4 \checkmark$
 $x_2 \rightarrow$ keine Lösung } $\mathbb{L} = \{2\}$

g) 1. Fall: $x - x = 0 = 4 \rightarrow$ keine Lösung

2. Fall: $-x - x = -2x = 4 \rightarrow x_1 = -2$

Probe: $x_1 = -2: |-2| - (-2) = 4 \checkmark \rightarrow$ $\mathbb{L} = \{-2\}$

h) 1. Fall: $x + 3 = 2x - 1 \rightarrow 4 = x_1$
 2. Fall: $-x - 3 = 2x - 1 \rightarrow 3x = -2 \rightarrow x_2 = -2/3$
 Probe: $x_1 = 4: |4 + 3| = 7 = 2 \cdot 4 - 1 \checkmark$
 $x_2 = -2/3: |-2/3 + 3| = 2 \cdot (-2/3) - 1 \rightarrow 7/3 = -7/3 \times$
 $\mathbb{L} = \{4\}$

i) 1. Fall: $3x - 2 = 2x + 12 \rightarrow x_1 = 14$
 2. Fall: $-3x + 2 = 2x + 12 \rightarrow 5x_2 = -10 \rightarrow x_2 = -2$
 Probe: $x_1 = 14: |3 \cdot 14 - 2| = 40 = 2 \cdot 14 + 12 \checkmark$
 $x_2 = -2: |3 \cdot (-2) - 2| = |-8| = 8 = 2 \cdot (-2) + 12 \checkmark$
 $\mathbb{L} = \{-2, 14\}$

j) 1. Fall: $5x - 8 = 3x - 16 \rightarrow 2x = -8 \rightarrow x_1 = -4$
 2. Fall: $-5x + 8 = 3x - 16 \rightarrow 8x = 24 \rightarrow x_2 = 3$
 Probe: $x_1 = -4: |5 \cdot (-4) - 8| = 28 = 3 \cdot (-4) - 16 = -28 \times$
 $x_2 = 3: |5 \cdot 3 - 8| = 7 = 3 \cdot 3 - 16 = -7 \times \Rightarrow \underline{\underline{\mathbb{L} = \{\}}}$

k) 1. Fall: $x + 11 = 3x - 13 \rightarrow 2x = 24 \rightarrow x_1 = 12$
 2. Fall: $-x - 11 = 3x - 13 \rightarrow 4x = 2 \rightarrow x_2 = 1/2$
 Probe: $x_1 = 12: |12 + 11| = 23 = 3 \cdot 12 - 13 \checkmark$
 $x_2 = 1/2: |1/2 + 11| = 11.5 = 3 \cdot (1/2) - 13 = -11.5 \times$
 $\mathbb{L} = \{12\}$

4a) 1. Fall: $2x + x = 3x = 12 \rightarrow x_1 = 4$
 2. Fall: $2x - x = x = 12 \rightarrow x_2 = 12$
 3. Fall: $-2x + x = -x = 12 \rightarrow x_3 = -12$
 4. Fall: $-2x - x = -3x = 12 \rightarrow x_4 = -4$
 Probe: $x_1 = 4: |2 \cdot 4| + |4| = 12 \checkmark$
 $x_2 = 12: |2 \cdot 12| + |12| = 36 \neq 12 \times$
 $x_3 = -12: |2 \cdot (-12)| + |-12| = 36 \neq 12 \times$
 $x_4 = -4: |2 \cdot (-4)| + |-4| = 8 + 4 = 12 \checkmark \Rightarrow \underline{\underline{\mathbb{L} = \{-4, 4\}}}$

- b) 1. Fall: $2x + x = 12 = 3x \rightarrow x_1 = 4$
 2. Fall: $2x - x = x = 12 \rightarrow x_2 = 12$
 3. Fall: $-2x - x = -3x = 12 \rightarrow x_3 = -4$
 4. Fall: $-2x + x = -x = 12 \rightarrow x_4 = -12$

Probe: $x_1 = 4: |2 \cdot 4 + |4|| = 12 \checkmark$
 $x_2 = 12: |2 \cdot 12 + |12|| = 36 \neq 12 \times$
 $x_3 = -4: |2 \cdot (-4) + |-4|| = 4 \neq 12 \times$
 $x_4 = -12: |2 \cdot (-12) + |-12|| = 12 \checkmark \Rightarrow \underline{\underline{\mathbb{L} = \{-12, 4\}}}$

- c) 1. Fall: $x + 11 + x + 5 = 2x + 16 = 10 \rightarrow x_1 = -3$
 2. Fall: $x + 11 - x - 5 = 6 \neq 10 \times$ keine Lösung!
 3. Fall: $-x - 11 + x + 5 = -6 \neq 10 \times$ " "
 4. Fall: $-x - 11 - x - 5 = -2x - 16 = 10 \rightarrow x_2 = -13$

Probe: $x_1 = -3: |-3 + 11| + |-3 + 5| = 8 + 2 = 10 \checkmark$
 $x_2 = -13: |-13 + 11| + |-13 + 5| = 2 + 8 = 10 \checkmark \rightarrow \underline{\underline{\mathbb{L} = \{-13, -3\}}}$

- d) 1. Fall: $x + 10 - (x - 10) = 20 \neq 10 \rightarrow$ keine Lösung!
 2. Fall: $x + 10 + x - 10 = 2x = 10 \rightarrow x_1 = 5$
 3. Fall: $-x - 10 - (x - 10) = -2x = 10 \rightarrow x_2 = -5$
 4. Fall: $-x - 10 + x - 10 = -20 \neq 10 \rightarrow$ keine Lösung!

Probe: $x_1 = 5: |5 + 10| - |5 - 10| = 15 - 5 = 10 \checkmark$
 $x_2 = -5: |-5 + 10| - |-5 - 10| = 5 - 15 = -10 \neq 10 \times$
 $\rightarrow \underline{\underline{\mathbb{L} = \{5\}}}$

- e) 1. Fall: $4x - 6 - x - 3 = 3x - 9 = 18 \rightarrow x_1 = 9$
 2. Fall: $4x - 6 + x + 3 = 5x - 3 = 18 \rightarrow x_2 = 4.2$
 3. Fall: $-4x + 6 - x - 3 = -5x + 3 = 18 \rightarrow x_3 = -3$
 4. Fall: $-4x + 6 + x + 3 = -3x + 9 = 18 \rightarrow x_4 = -3$

Probe: $x_1 = 9: 2 \cdot |2 \cdot 9 - 3| - |9 + 3| = 18 \checkmark$
 $x_2 = 4.2: 2 \cdot |2 \cdot 4.2 - 3| - |4.2 + 3| = 3.6 \neq 18$
 $x_3 = -3: 2 \cdot |2 \cdot (-3) - 3| - |-3 + 3| = 2 \cdot |-9| + 0 = 18 \checkmark$
 $x_4 = x_3 \rightarrow \underline{\underline{\mathbb{L} = \{-3, 9\}}}$

f) 1. Fall: $(x+1) \cdot (x-1) = x^2 - 1 = 8 \rightarrow x = \pm 3$
 2. Fall: $(x+1) \cdot (1-x) = 1 - x^2 = 8 \rightarrow x^2 = -7 \rightarrow$ keine Lösung!
 Probe: $x_1 = 3: (3+1) \cdot |3-1| = 4 \cdot 2 = 8 \checkmark$
 $x_2 = -3: (-3+1) \cdot |-3-1| = (-2) \cdot 4 = -8 \neq 8 \times$
 $\rightarrow \underline{\underline{\mathbb{L} = \{3\}}}$

g) 1. Fall: $\frac{x+12}{x} = \frac{3x}{x} \rightarrow 12 = 2x \rightarrow x_1 = 6$
 2. Fall: $\frac{x+12}{-x} = \frac{-3x}{-x} \rightarrow 12 = -4x \rightarrow x_2 = -3$
 Probe: $x_1 = 6: \left| \frac{6+12}{6} \right| = \frac{18}{6} = 3$
 $x_2 = -3: \left| \frac{-3+12}{-3} \right| = \left| \frac{9}{-3} \right| = 3$

h) 1. Fall: $4x + 8 = 3x + 27 \rightarrow x_1 = 19$
 2. Fall: $-4x - 8 = 3x + 27 \rightarrow 7x = -35 \rightarrow x_2 = -5$
 3. Fall: $4x + 8 = -3x - 27 \rightarrow 7x = -35 \rightarrow x_3 = -5$
 4. Fall: $-4x - 8 = -3x - 27 \rightarrow -7x = -19 \rightarrow x_4 = 19$
 Probe: $x_1 = x_4 = 19: 2 \cdot |2 \cdot 19 + 4| = 3 \cdot |19 + 9| \rightarrow 84 = 84 \checkmark$
 $x_2 = x_3 = -5: 2 \cdot |2 \cdot (-5) + 4| = 3 \cdot |-5 + 9| \rightarrow 12 = 12 \checkmark$
 $\rightarrow \underline{\underline{\mathbb{L} = \{-5, 19\}}}$

5.)

x	y ₁	y ₂
-10	5	15
-8	3	13
-6	1	11
-4	-1	9
-2	-3	7
0	-5	5
2	-7	3
4	-9	1
6	-11	-1
8	-13	-3
10	-15	-5

